Exercise 19

Find the limit or show that it does not exist.

$$\lim_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2}$$

Solution

Multiply the numerator and denominator by the reciprocal of the highest power of t in the denominator.

$$\begin{aligned}
\lim_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2} &= \lim_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2} \cdot \frac{\frac{1}{t^2}}{\frac{1}{t^2}} \\
&= \lim_{t\to\infty} \frac{(\sqrt{t}+t^2)\frac{1}{t^2}}{(2t-t^2)\frac{1}{t^2}} \\
&= \lim_{t\to\infty} \frac{\frac{1}{t^{3/2}}+1}{\frac{2}{t}-1} \\
&= \frac{\lim\limits_{t\to\infty}\left(\frac{1}{t^{3/2}}+1\right)}{\lim\limits_{t\to\infty}\left(\frac{2}{t}-1\right)} \\
&= \frac{\lim\limits_{t\to\infty}\frac{1}{t^{3/2}}+\lim\limits_{t\to\infty}1}{\lim\limits_{t\to\infty}\frac{2}{t}-\lim\limits_{t\to\infty}1} \\
&= \frac{0+1}{0-1} \\
&= -1
\end{aligned}$$